# DAMPING OF STRONG SHOCKS IN LAMINAR MATERIALS

V. F. Nesterenko, V. M. Fomin, and
P. A. Cheskidov

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The investigation of strong shock propagation processes in laminar media has two aspects. The phenomenon of pressure, and mass flow rate attenuation or magnification on the wave front as a function of the set of the laminar system should be referred to the first. It was predicted in [1] that as the layer thickness increased for systems of alternating plane light- and heavy-material layers the phenomenon of unlimited accumulation can be obtained. This phenomenon was later studied numerically and experimentally in [1-4]. Formulas permitting the determination of the pressure and mass flow rate behind a bow shock front are obtained in [5-6] in a linear approximation, as it moves over the laminar material. It is shown first here that an increase or decrease in the pressure and mass flow rate are connected single-valuedly with the change in the acoustic impedances of the layers. This fact was later confirmed even for nonlinear interaction just for media consisting of two and three different layers [5-8]. However, it was noted in [9] that the growth in amplitude of the bow shock pressure is observed not only because of dissociation of the discontinuity on the boundary of materials with different acoustic stiffness, but also because of the origination of compression waves that overtake the bow wave.

A shock with pressure varying periodically in the front can be propagated during steady motion in a periodic laminar system [1]. The cell dimension here determines just the scale of the phenomenon, and the build-up time of the stationary wave pattern, and does not influence the pressure amplitude. It was shown experimentally in [10] that the bow wave amplitude at the identical depth is independent of the number of cells in the periodic laminar material (LM). Application of the approach developed in [5] yields strong damping of the bow wave amplitude as the number of crossings of the interfaces increases. This is associated with not taking account of nonlinear effects. On the basis of an analysis of the nonlinear wave equation it is shown in [11, 12] that an experimentally verified increase occurs in the amplitude of a weak shock as it is propagated in a periodic LM.

The investigations [10, 13, 14] that set up when heterogeneous media can be modeled by laminar systems should be referred to the second aspect.

Despite the large quantity of papers on the investigation of shocks in laminar materials, propagation of finite-duration waves in periodic LM during the unsteady motion phase with nonlinear effects taken into account, which can result in qualitatively new phenomena not described by linear models, has not been studied in practice at this time.

The process of strong shock damping in periodic LM under impulsive loading in the unsteady motion phase is investigated numerically and experimentally in this paper. It is shown that the influence of the loading waves overtaking the bow wave can play a governing role in the change in bow wave amplitude. The influence of the number of cells, and the wave amplitude and length on the nature of its damping is investigated.

## 1. Physicomathematical Formulation of the Problem and Method of Solving It

Let us consider the problem of strong shock propagation in a laminar material consisting of n identical cells, each of which contains a layer of the first substance of thickness $h_1$ and of the second of thickness $h_2$. We consider the layer thickness much less than the diameter of the LM, which permits solving the problem in a one-dimensional approximation. The LM is loaded analogously to [15]. At a certain time a charge on the free surface, in contact with the LM to be loaded, is excited by a detonation wave moving at the velocity D. During the time $t_0 = H/D$, where H is the charge thickness, it reaches the boundary of contact with the LM and communicates a dynamic load thereto.

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TABLE 1

| Material | $\rho_0$, mg/mm$^3$ | $\gamma$ | A, GPa | $c_0$, mm/μsec | $\lambda$ |
|---|---|---|---|---|---|
| Teflon-4 | 2,19 | 6,9 | 1,005 | 1,98 | 1,71 |
| Paraffin | 0,9 | 4,1 | 2,394 | 3,3 | 1,31 |
| Aluminum | 2,787 | 4,2 | 19,3 | 5,25 | 1,39 |
| Copper | 8,9 | 4,8 | 29,6 | 3,95 | 1,50 |

We describe the behavior of the solid body and the detonation products (DP) by a hydrodynamic model in the Euler coordinates (t, x), which has the following form in the case of one-dimensional flows with plane symmetry

$$d\rho/dt + \rho\partial u/\partial x = 0,\quad \rho du/dt + \partial p/\partial x = 0,\quad de/dt + pdv/dt = 0, \tag{1.1}$$

where $\rho$ is the density of the substance, p is the pressure, e is the specific internal energy, u is the velocity, v is the specific volume defined by the relationship $v = 1/\rho$; $d/dt = \partial/\partial t + u\partial/\partial x$. We close the system (1.1) with the equation of state

$$p = p(\rho, e). \tag{1.2}$$

At the time t = 0 let a detonation wave emerge on the contact surface between the charge and the laminar material. We denote the running coordinate of the DP free surface by $l_0$, the free LM surface by $l_{2n+1}$, and its contact surfaces by $l_i$, i = 1,..., 2n. Then this problem is formulated mathematically as follows: Find the functions u, p, $\rho$, e in the domain $Z = \{l_0(t) < x < l_{2n+1}(t),\ 0 < t < \infty\}$, that satisfy the system of differential equations (1.1) and (1.2) with the following initial and boundary conditions for $l_i(t) < x < l_{i+1}(t)$ (i = 0,..., 2n)

Initial Conditions. The distribution of the DP parameters behind the detonation wave front at the time t = 0 is given from the self-similar solution describing a Chapman—Jouget detonation wave [16], and we assume in the LM at t = 0

$$p = 0,\ u = 0,\ \rho = \rho_i\ \text{for}\ l_i < x < l_{i+1},\ i = 1, \ldots, 2n.$$

Boundary Conditions. For $t \geqslant 0$ the pressure is given equal to zero on the free surfaces $x = l_0(t)$ and $x = l_{2n+1}(t)$, and compliance with the continuity conditions for the normal stresses and velocities is required for the contact surfaces $x = l_i(t)$ (i = 1,..., 2n).

The problem formulated was solved numerically by using a Wilkins-type through-computation difference scheme [17] in which artificial viscosity was used for stable computation of the compression waves. Linear viscosity

$$q = \begin{cases} -q_0 h c_0 \rho_0 \partial u/\partial x & \text{for}\ \partial u/\partial x < 0, \\ 0 & \text{for}\ \partial u/\partial x \geqslant 0, \end{cases}$$

was taken in the computations, where h is the mesh spacing, and $q_0$ is a constant [18].

Methodological questions associated with singularities of the numerical solution of the problem formulated by the mentioned finite-difference method are considered [9, 15].

In the numerical computations the equation (1.2) for the detonation products is selected in the form $p = (\gamma - 1)\rho e$, and the computations were performed for the materials comprising the LM for comparison with the two equations

in the Tait form

$$p = A((\rho/\rho_0)^\gamma - 1); \tag{1.3}$$

in the form

$$p = \frac{c_0^2\rho_0\left(1 - \frac{\rho_0}{\rho}\right)}{\left(\left(\frac{\rho_0}{\rho} - 1\right)\lambda + 1\right)^2}, \tag{1.4}$$

obtained from the dependence $D = c_0 + \lambda u$, where D is the shock velocity, u is the mass flow rate, and $c_0$, $\lambda$ are constants. The magnitudes of the parameters in (1.3) and (1.4) are presented in the table.

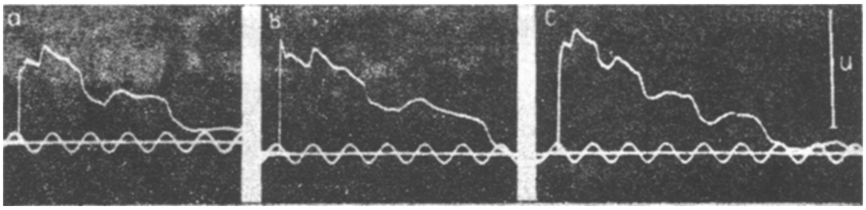


Fig. 1

## 2. Methodology of Performing the Experiment

Shocks were produced in all the materials under investigation by using a plane shock generator [15].

Measurement of the mass flow rate in the Teflon—paraffin, LM, which remains a dielectric under the pressures being investigated, was by an electromagnetic method [19]. Values of the velocities were found from the formula $u = \varphi 10^8/(H_1 d)$, where $H_1 = 396$ Oe is the magnitude of the permanent magnetic field intensity used in the experiment, d is the sensor crossbar width equal to 7.6 mm, and $\varphi$ is the voltage on the sensor as it moves.

The laminar Teflon—paraffin material was assembled from individual paraffin and Teflon plates glued together by glue No. 88. The specimen was a cylinder of diameter 40 mm. Loading was along the cylinder axis so that the plane of the shock was parallel to the plane of the plates comprising the LM. The mean density of the specimens was checked in the tests. The measuring sensor was placed on one of the contact surfaces. Typical oscillograms for the mass flow rate measured for a LM with the cell dimension $\delta = 5$ mm between cells 2 and 3 (curve *a*) and 5 and 6 (curve b) and for a LM with $\delta = 2.5$ mm between cells 10 and 11 (curve c), are presented in Fig. 1. The frequency of the timing mark is 1 mHz. The scale with u = 1 mm/μsec amplitude refers to curves b and c, while for curve *a* it is twice as large.

The laminar material Al—Cu was loaded by an analogous method. The diameter of specimens comprised of Al and Cu plates was 50 mm. Measurement of the LM Al—Cu free surface velocity was by a contactless method [15]. This method is based on recording the electromagnetic perturbations occurring during motion of the free metal surface in the electromagnetic field. The axis of the cylindrical specimen is placed perpendicularly to the vector $H_1$ in this case. The shock was also propagated along the specimen axis. In this case a contour of several turns (1-10) of copper wire of diameter 0.1 mm wound on an organic glass holder of dimensions 1.25 × 6 mm. The magnitude of the signal depends on the specimen geometry and the loading conditions in this measurement scheme. A linear dependence of the emf being measured in the contour was found by special tests with copper specimens in an analogous geometry from the flight velocity of the central part (diameter 20 mm) of the free surface with coefficient 60 ±4 mV/(mm/μsec). In our geometry, a constant emf for ∿5 μsec corresponds to a constant flight velocity. The scheme assures good time resolution, is interference-immune. The means velocity of the free surface on a 3-4.5-mm base was also checked in all tests with this methodology by the times of the beginning of free surface motion and the beginning of its deceleration by the glass block placed in front of the measuring element. This check aids in verification of the deductions obtained on the basis of a continuous velocity measurement.

## 3. Analysis of the Computation and Experiment Results

Upon loading a LM by detonation products, a head wave is propagated, behind which a complex wave pattern that varies with the lapse of time, is formed because of compression and rarefaction wave interaction and with the contact boundaries. The head wave also varies with time: first, because of the dissociation of the discontinuities on the contact interfaces of the different materials in the LM, whereupon the formation occurs of waves moving in the forward and reverse directions; second, because of interaction with the waves that overtake the head wave moving at a higher velocity over the preliminarily loaded material. It is interesting to study the influence of both the first and the second factors on the nature of the change in head wave amplitude.

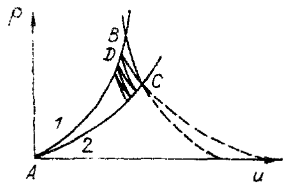


Fig. 2

Without taking account of the overtaking effects, the influence of the first factor is conveniently demonstrated on the p—u-diagram (Fig. 2). Let the curve 1 be the shock polar of the first substance, and curve 2 of the second substance. The point A corresponds to states before the head wave front, where the material is not loaded. Let the shock move over the layer of the first substance, and let the point B correspond to the state behind the wave front. After a certain time the shock will reach the interface between the first and second substances. A refracted shock, whose state behind the front is marked by the letter C, and a reflected shock or rarefaction wave depending on the mutual location of curves 1 and 2 are formed because of interaction between the shock and the contact boundaries. After interaction between the head wave and the next contact boundary (now between the second substance and the first) the state marked by the point D, which lies below the point B is formed behind the front. The effect mentioned occurs because of the discrepancy in the values of the acoustic impedances of the substances from which the layers consist. The more strongly the acoustic impedances of the layers are distinguished, the lower will the point B lie on the shock polar relative to D, meaning that the stronger will the pressure and mass flow rate amplitude in the head wave drop. A further pattern of the change in head wave amplitude is seen in Fig. 2. The different laminar vibration dampers and antimeteorite shields [20] are based on this property.

Interaction between the head and unloading wave that occurs because of escape of the detonation products into a vacuum, resulting in constant head wave damping, and between the head and the load reflected waves overtaking the head wave, should be referred to the second factor.

The influence of both factors on the nature of the change in head wave amplitude was estimated in the example of the LM Teflon-4—paraffin (Ф — П) and aluminum-copper (Al—Cu). The ratio of the acoustic impedances is 1.46 in the first case, and 2.38 in the second. The charge thickness was 16.8 mm in the computations, which took into account the influence of the planewave lens. The material parmeters and numerical values of the model constants are presented in the table. The data are obtained on the basis of [21, 22]. The results of computations performed with the equation of state in the form (1.3) and (1.4) are practically in agreement.

It must be noted that, strictly speaking, repeated shock loading in laminar materials should be described by multiple-compression shock adiabats. In our computations the multiple-

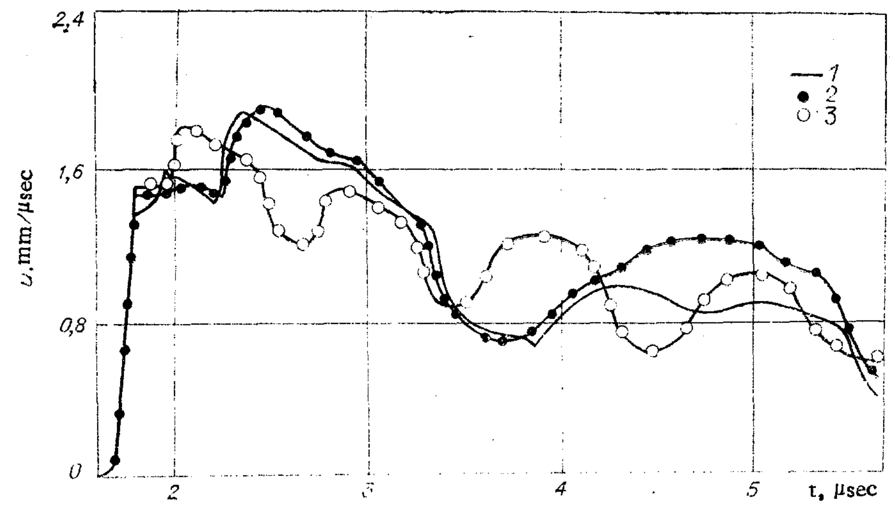


Fig. 3

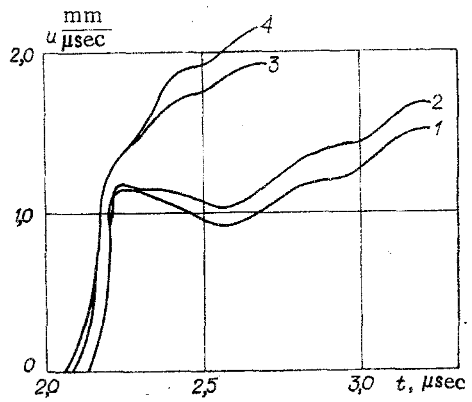

Fig. 4

compression shock adiabat was considered to agree with the single-compression shock adiabat, which is apparently justified for finding the mechanical shock parameters of tens of gigapascals in intensity because of the small contribution of the thermal pressure as compared with the cold compression pressure. The good agreement between the numerical computations and experiment, as is observed even in such strongly compressible materials as paraffin and Teflon, is indirect confirmation of the validity of such an approach.

To clarify the influence of different factors on the nature of the change in amplitude of the head wave, experimental investigations and numerical computations were performed on composites of a single common length, but with a different magnitude cell, the quantity of the contact boundaries in the LM was thereby variated.

The experimental and computed time dependences of the mass flow rate in the material Ф—П are displayed in Fig. 3. The total length of the LM was 50 mm. The Teflon plate was located closest to the charge. The magnitude of the cell $\delta$ was 5 and 2.5 mm in the numerical computations. The ratio between the layer thicknesses was $h_1/h_2 = 3/7$. The dependences were taken off on the boundary between cells 2 and 3 in a LM with $\delta = 5$ mm, which corresponded to L = 10 mm from the boundary of the contact with the DP. Comparison of the dependences obtained numerically and experimentally (curves 2 and 1 in Fig. 3, respectively) for $\delta = 5$ mm, shows their good quantitative and qualitative agreement in complex wave process details. A certain deviation of the computed dependence from the experimental for times greater than 4.5 μsec is visibly related to the influence of side unloading, i.e., in this case the physical process becomes substantially nonuniform. It is seen in Fig. 3 that the amplitudes of the head wave mass flow rate in LM with $\delta = 5$ and 2.5 mm (curve 3 in Fig. 3) agree within the limits of accuracy of the computations, although in the second case the head wave encountered twice as many contact surfaces in its path. The same result is obtained in experiments to record the time dependences of the mass flow rate at the depth L = 25 mm in LM with $\delta = 2.5$ and 5 mm, and in numerical computations at a depth L = 25 mm with $\delta = 2.5$, 5, and 25 mm, as well as a depth of L = 40 mm in a LM with $\delta = 2.5$ and 5 mm. This phenomenon contradicts the essential approach to the consideration of shockwave processes in laminar materials that is based solely on the interaction between the head wave and the interfaces and yields strong damping of the head wave amplitude as the number of crossings of the interfaces increases.

An analogous result is obtained numerically in the problem of loading the material Ф—П with the cell $\delta = 25$ and 2.5 mm by a flat impactor that produces a wave with parameters close to the DP loading. The thickness of the impactor was selected to be adequate to eliminate the influence of the rarefaction wave from its free surface on the motion of the contact boundary $l_1$, meaning also the wave pattern in the LM. Therefore, independence of the head wave amplitude from the number of crossings of the interfaces at a given depth is not related to the influence of the rarefaction wave but to the nonlinear effects of the overtakings.

Analogous investigations were performed on the LM Al—Cu with the cell magnitudes $\delta = 4$ and 2 mm. Each cell contained an aluminum plate and a copper plate of identical thickness. The aluminum layer was closest to the charge. The total LM length was 12 mm in the experiments, and 12 and 16 mm in the numerical computations.

Velocity profiles of the free surface vs time in the LM Al—Cu are presented in Fig. 4, where curve 1 is a computation, 2 is an experiment for $\delta = 4$ mm, curve 3 a computation, and

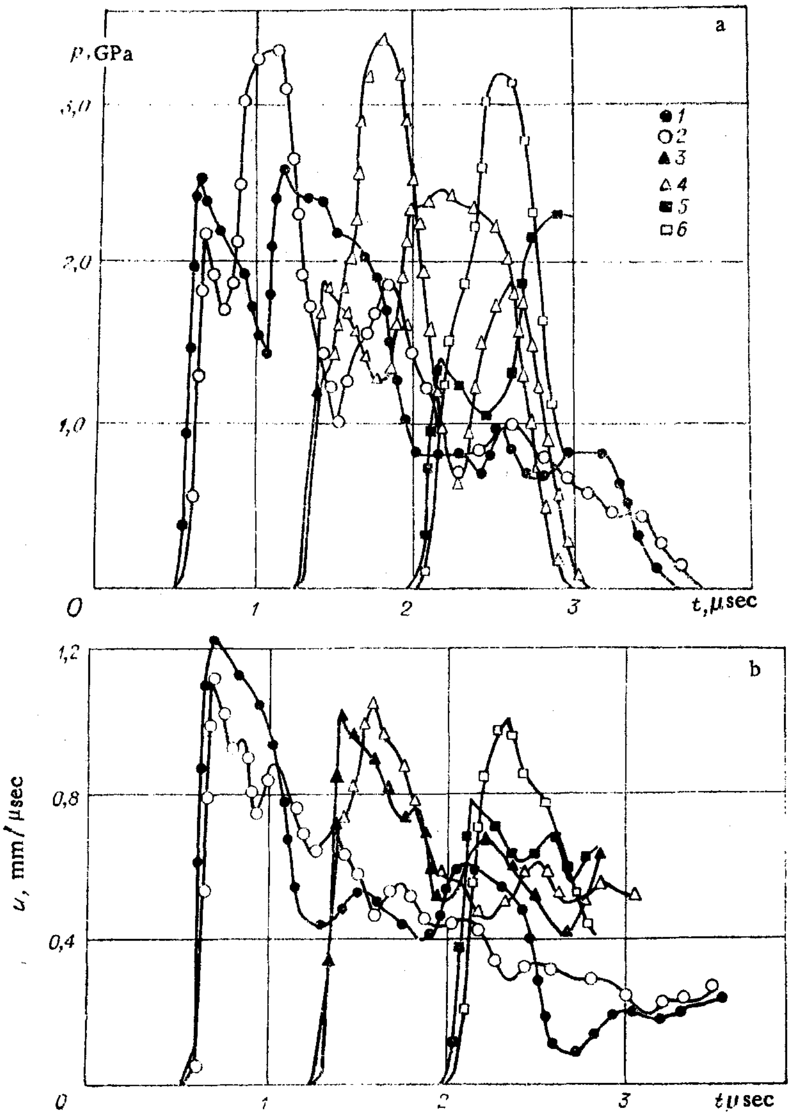


Fig. 5

4 an experiment for δ =2 mm. The computational and experimental dependences are sufficiently close to each other, and quantitative and qualitative agreement on the wave patterns of the phenomenon holds. The velocity profile of the free surface has a three-wave configuration for both δ =4 mm and δ =2 mm. The amplitude of the second and third waves is somewhat lower than in experiments. This is visibly explained by the possibility of the plates in the experiment recoiling from each other and their subsequent snapping. It is unexpected that the amplitude of the head wave in the LM with the finer cell is higher than in the LM with the large cell. Analysis of the p—u-diagram without taking account of the influence of the overtaking load waves on the head wave shows that the amplitude of the head wave mass flow rate in the LM Al—Cu with δ =2 mm should be almost half that in the LM with δ =4 mm. It is especially interesting that this phenomenon is obtained for loading by a triangular pressure pulse. This result indicates the governing role of the overtaking effects associated with the nonlinearity in material behavior, in head wave formation.

Displayed in Figs. 5a and b are computational time dependences of the pressure and mass flow rate at different depths in LM Al—Cu (curve 1) L =4 mm, δ =4 mm; 2) L =4 mm, δ =2 mm; 3) L =8 mm, δ =4 mm; 4) L =8 mm, δ =2 mm; 5) L =12 mm, δ =4 mm; 6) L =12 mm, δ =2 mm). Clearly seen in these graphs is the dynamics of wave formation and the influence of the various factors on the nature of the change in the head wave amplitude. Thus, we see a significant diminution in the amplitude of the head wave as the number of crossings of the interfaces increases at the depths L =4 and 8 mm, where the influence of the overtaking effects is still not felt. In this domain of the distances traversible by a shock of given amplitude

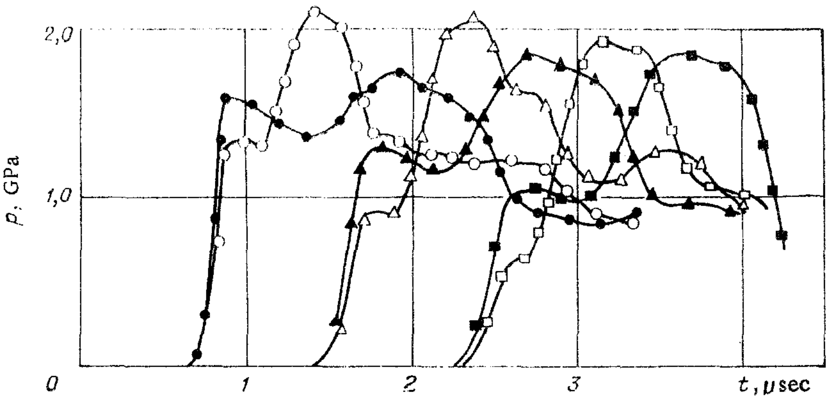


Fig. 6

and duration, the pressure and velocity can be determined behind its front on the basis of a simple computation in the p—u-diagram if the damping effects are not essential because of the rarefaction wave. Consequently, a computation of the effects of cumulation in inhomogeneous laminar materials, based on the examination of p—u-diagrams [5], is possible for a small number of layers. At the same time the extension of this analysis to a large number of layers raises doubts. Indeed, even at an 8-mm depth in a LM with $\delta = 2$ mm it is clearly seen that a second loading wave moves behind the head wave, and overtakes the first and substantially increases the mass flow rate and pressure behind it. At a 12-mm depth the head wave amplitude in a LM with $\delta = 2$ mm is already considerably greater than in a LM with $\delta = 4$ mm. In addition, a comparison of the maximal values of the pressure shows that it is 40% higher in the LM Al—Cu with $\delta = 2$ mm than in the LM with $\delta = 4$ mm.

It is interesting to study the influence of the wave amplitude on the nature of its damping since the role of the overtaking effects for weak shocks at the same distances should be lowered to a significant extent because of the diminution in the nonlinearity of the material behavior. Presented in Fig. 6 are time dependences of the pressure in a LM Al—Cu under a loading by a weaker triangular pulse (same notation as in Fig. 5). Firstly the strong damping of the head wave amplitude with the increase in the number of interface crossings, and secondly the absence of anomalously high values of the pressure in the LM with a finer cell can here be noted.

Therefore, the nonlinear overtaking effects, resulting in qualitative singularities distinct from the linear analysis, appear more substantially for strong waves of finite duration with a 6-7 cell spatial scale. It can be expected that as the loading wave duration diminishes even though its amplitude is high, the effects considered will appear weaker. In particular, if the spatial size of the compression pulse is less than the size of one plate in the LM, then the overtaking effects cannot generally appear, and the damping of even a strong shock can be estimated on the basis of an examination of the interaction between just the head wave and the contact surfaces, and of the influence of rarefaction wave thereon. We performed computations for a shock produced in a LM Al—Cu by detonation

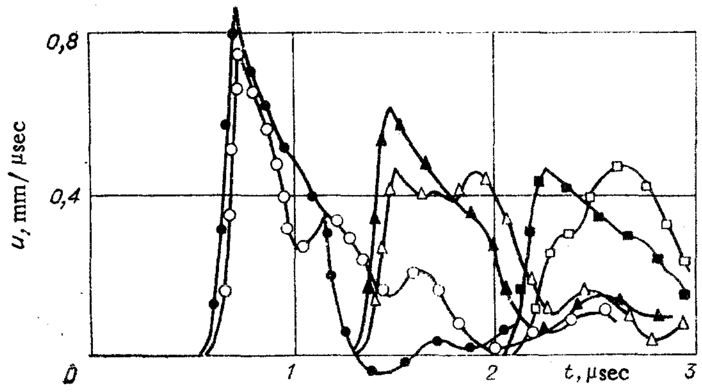


Fig. 7

of a layer of the same charge but with a 4.2-mm thickness (Fig. 7; same notation as in Fig. 5). From a comparison of the results presented in Figs. 5 and 7 it is seen that the effect mentioned actually takes place. In particular, for L = 12 mm the head wave in both a LM with $\delta = 2$ mm, and a LM with $\delta = 4$ mm, is clearly isolated in Fig. 7, where the amplitude in the LM with the finer cell is considerably lower than in the LM with the coarse cell.

Numerical computations in a LM with Al—Cu were performed by using an elastic—plastic model also. The qualitative results elucidated above agree for the computations with the hydrodynamic and elastic—plastic models.

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# RESPONSE OF AN ANHARMONIC CRYSTAL TO A LOCALIZED INITIAL IMPETUS

A. S. Dolgov

UDC 534+539.2

There is much interest in the time dependence of the atomic displacements in a crystal after an initial impetus because of the need to examine the interactions of an atomic-particle beam with a solid surface, which includes nonstationary deformation after localized action of impact type. There are several papers dealing with the response of crystals to external shocks or with discussion of the physical effects associated with the response function [1-4]. Nearly all theoretical papers on this topic employ the harmonic approximation. The role of slight anharmonicity has been discussed in [5].

However, most of the physical processes involved here are based on levels of initial excitation such that one cannot assume that anharmonic effects are small or unimportant. Therefore, major interest attaches to proper incorporation of the nonlinearity in the interaction. Numerical calculations, although useful, are only partial in character and cannot completely replace analytical consideration designed to elucidate the general features. Here we present a certain class of solutions for the displacements in a decidedly nonlinear structure.

We take a structure with a power-law dependence for the potential energy on the relative displacements, which under certain conditions given below allows one to determine the main features in the process. We give the following form to the equations of dynamics for a one-dimensional atomic chain with interaction between nearest neighbors:

$$d^2x_n/dt^2 = \alpha\{(x_{n-1} - x_n)^{2p+1} - (x_n - x_{n+1})^{2p+1}\}, \tag{1}$$

where $x_n$ is the displacement of an atom, which is assigned subscript n, relative to its equilibrium position, while p is an integer that is not zero (subject to certain reservations, the constructions given below can be extended to the case of arbitrary nonnegative value of p). Equation (1) does not contain a linear component. This feature of the force interaction occurs for example in the transverse component of the vibrations in a rectilinear atomic chain, where the lowest order in the dependence of the forces on the displacements corresponds to the third degree. Also, the role of the linear component may be secondary for other structures with vibrations of large scale. Of course, incorporation of the components linear in the displacements would extend the range of real objects that correspond qualitatively to (1), but in that case one does not obtain clear final formulas. On the other hand, solutions in the form of long-wave solitons that can be derived are of other interest, but the purposes differ from those of this study. Therefore, we take (1) as the starting point.

The continuum approximation for (1) gives

$$\frac{\partial^2 x}{\partial t^2} = \alpha \frac{\partial}{\partial n}\left(\frac{\partial x}{\partial n}\right)^{2p+1}. \tag{2}$$

Considerations of scale invariance for (2) suggest that it is desirable to seek the solutions in the form

$$x(n,\ t) = f(\xi), \quad \xi = nt^{-1/(p+1)}. \tag{3}$$

Substitution of (3) into (2) reduces the latter to an ordinary differential equation:

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